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Abstracts of Papers to Appear

A Multidomain Spectral Method for Scalar and Vectorial Poisson Equations with Noncompact Sources. P. Grandclément, S. Bonazzola, E. Gourgoulhon, and J.-A. Marck. Département d'Astrophysique Relativiste et de Cosmologie (UMR 8629 du C.N.R.S.), Observatoire de Paris, Section de Meudon, F-92195 Meudon Cedex, France.

We present a spectral method for solving elliptic equations which arise in general relativity, namely three-dimensional scalar Poisson equations, as well as generalized vectorial Poisson equations of the type $\Delta\vec{N} + \lambda\vec{\nabla}(\vec{\nabla}\cdot\vec{N}) = \vec{S}$ with $\lambda \neq -1$. The source can extend in all the Euclidean space $\mathbf{R}^3$, provided it decays at least as r^{-3}. A multidomain approach is used, along with spherical coordinates (r, θ, ϕ). In each domain, Chebyshev polynomials (in r or $1/r$) and spherical harmonics (in θ and ϕ) expansions are used. If the source decays as r^{-k} the error of the numerical solution is shown to decrease at least as $N^{-2(k-2)}$, where N is the number of Chebyshev coefficients. The error is even evanescents; i.e., it decreases as $\exp(-N)$, if the source does not contain any spherical harmonics of index $l \geq k - 3$ (scalar case) or $l \geq k - 5$ (vectorial case).

Unconditionally Stable Finite Difference Scheme and Iterative Solution of 2D Microscale Heat Transport Equation. Jun Zhang* and Jennifer J. Zhao.† *Laboratory for High Performance Scientific Computing and Computer Simulation, Department of Computer Science, University of Kentucky, 773 Anderson Hall, Lexington, Kentucky 40506-0046; and †Department of Mathematics and Statistics, University of Michigan at Dearborn, Dearborn, Michigan 48128-1491.

A two-dimensional time-dependent heat transport equation at the microscale is derived. A second order finite difference scheme in both time and space is introduced and the unconditional stability of the finite difference scheme is proved. A computational procedure is designed to solve the discretized linear system at each time step by using a preconditioned conjugate gradient method. Numerical results are presented to validate the accuracy of the finite difference scheme and the efficiency of the proposed computational procedure.

A Local Support-Operators Diffusion Discretization Scheme for Hexahedral Meshes. J. E. Morel, Michael L. Hall, and Mikhail J. Shashkov. University of California, Los Alamos National Laboratory, Los Alamos, New Mexico 87545.

We derive a cell-centered 3-D diffusion differencing scheme for unstructured hexahedral meshes using the local support-operators method. Our method is said to be local because it yields a sparse matrix representation for the diffusion equation, whereas the traditional support-operators method yields a dense matrix representation. The diffusion discretization scheme that we have developed offers several advantages relative to existing schemes. Most importantly, it offers second-order accuracy on reasonably well-behaved nonsmooth meshes, rigorously treats material discontinuities, and has a symmetric positive-definite coefficient matrix. The order of accuracy is demonstrated computationally rather than theoretically. Rigorous treatment of material discontinuities implies that the normal component of the flux is continuous across such discontinuities while the parallel components may be either continuous or discontinuous in accordance with the exact solution to the problem being considered. The only disadvantage of the method is that it has both cell-centered and face-centered scalar unknowns as opposed to just cell-center scalar unknowns. Computational examples are given which demonstrate the accuracy and cost of the new scheme.

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Stability of High-Order Perturbative Methods for the Computation of Dirichlet–Neumann Operators. David P. Nicholls and Fernando Reitich. School of Mathematics, University of Minnesota, Minneapolis, Minnesota 55455.

In this paper we present results on the stability of perturbation methods for the evaluation of Dirichlet–Neumann operators (DNO) defined on domains that are viewed as complex deformations of a basic, simple geometry. In such cases, geometric perturbation methods, based on variations of the spatial domains of definition, have long been recognized to constitute efficient and accurate means for the approximation of DNO and, in fact, several numerical implementations have been previously proposed. Inspired by our recent analytical work, here we demonstrate that the convergence of these algorithms is, quite generally, limited by numerical instability. Indeed, we show that these standard perturbative methods for the calculation of DNO suffer from significant ill-conditioning which is manifest even for very smooth boundaries, and whose severity increases with boundary roughness. Moreover, and again motivated by our previous work, we introduce an alternative perturbative approach that we show to be numerically stable. This approach can be interpreted as a reformulation of classical perturbative algorithms (in suitable independent variables), and thus it allows for both direct comparison and the possibility of analytic continuation of the perturbation series. It can also be related to classical (preconditioned) spectral approaches and, as such, it retains, in finite arithmetic, the spectral convergence properties of classical perturbative methods, albeit at a higher computational cost (as it does not take advantage of possible dimensional reductions). Still, as we show, an alternative approach such as the one we propose may be mandated in cases where substantial information is contained in high-order harmonics and/or perturbation coefficients of the solution.

Retrieving the Balanced Winds on the Globe as a Generalized Inverse Problem. Huei-Iin Lu* and Franklin R. Robertson.† *Universities Space Research Association, Huntsville, Alabama; and †Marshall Space Flight Center, Huntsville, Alabama.

A generalized inverse technique is applied to retrieve two types of balanced winds that characterize the large-scale dynamics of the atmosphere: rotational winds based upon the linear balance equation and divergent winds based upon the vorticity budget equation. Both balance equations are singular at or near the equator. The balance equations are transformed in spherical harmonic function space to an underdetermined system, for which the scale-weighted, least-squares solution consists of a sum of principal and singular components. The principal components represent the response to the source function for the regular eigenmodes, while the singular components are determined by the projection of an independent measurement on the singular eigenmodes. The method was tested with the NCEP/NCAR reanalysis data in which a quasi-balance condition exists. A realistic balanced wind field is retrievable when the singular components are computed based upon the reanalyzed wind data.